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Partial solution by F. L. Wilmer, Omaha, Neb.

As usual let a and b denote the abscissa and ordinate, respectively, of the terminus of the vector $a + ib$ in its standard position, and $|a + ib|$ its absolute value.

Because $a^2 + b^2$ must be equal to the square of the cosine of some angle, say α, therefore $c^2 + d^2 = \sin^2 \alpha$. Similarly for x, y, z, w and an angle, say β.

For given values of the moduli the sum of any number of complex numbers has its maximum absolute value when the arguments are equal or differ by multiples of 2π, and this maximum is the sum of the moduli.

$$\begin{array}{lcccc} \text{Let the arguments of} & a + ib, & i(c + id), & x + iy, & i(z + iw), \\ \text{be} & \theta_1\ , & \theta_1'\ , & \theta_2\ , & \theta_2'\ . \end{array}$$

The arguments of the three terms of the given expression will be $\theta_1 + \theta_2$, $\theta_1 + \theta_2'$, $\theta_1' + \theta_2$, and these will be equal if $\theta_1' = \theta_1$ and $\theta_2' = \theta_2$ (to multiples of 2π).

The modulus of the given expression will then be

$$\begin{aligned} R &= 2 \cos \alpha \cos \beta + \cos \alpha \sin \beta + \sin \alpha \cos \beta \\ &= \cos (\alpha + \beta) + \cos (\alpha - \beta) + \sin (\alpha + \beta). \end{aligned}$$

Now in order that R be a maximum we must have $\partial R/\partial \alpha = 0$ and $\partial R/\partial \beta = 0$, simultaneously. That is,

$$- \sin (\alpha + \beta) - \sin (\alpha - \beta) + \cos (\alpha + \beta) = 0,$$

and

$$- \sin (\alpha + \beta) + \sin (\alpha - \beta) + \cos (\alpha + \beta) = 0;$$

whence $\alpha = \beta = \pi/8$, giving[1]

$$R_m = \cos \pi/4 + 1 + \sin \pi/4 = \sqrt{2} + 1.$$

2896. (1921, 227) **Proposed by the late L. G. WELD.**

A circle is inscribed in a triangle. In each of the three spandrels between this circle and the vertices another circle is described; in each of the three spandrels between the last circles and the vertices another circle; and so on *ad infinitum*. Show that the ratio of the sum of the areas of all the circles to the area of the triangle is

$$\frac{\Sigma}{\Delta} = \frac{\pi \Delta}{4 S^2} \left[\frac{1}{\sin \frac{1}{2}A} + \frac{1}{\sin \frac{1}{2}B} + \frac{1}{\sin \frac{1}{2}C} - 2 + \sin \tfrac{1}{2}A + \sin \tfrac{1}{2}B + \sin \tfrac{1}{2}C \right].$$

This problem is identical with problem 483 Geometry already proposed by Professor Weld (*1916*, 79), and solved by J. A. Caparo (*1916*, 344–346). It was re-proposed through an oversight.

NOTES AND NEWS.

It is to be hoped that readers of the MONTHLY will coöperate in contributing to the general interest of this department by sending items to H. P. MANNING, Brown University, Providence, R. I.

At the University of Pennsylvania, Mr. J. D. Eshleman, of the University of Chicago, has been appointed instructor of mathematics. Mr. J. M. Thomas resigned his instructorship to accept a Harrison Fellowship in the Graduate School.

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[1] We might say, $R = \sqrt{2} \cos (\pi/4 - \alpha - \beta) + \cos (\alpha - \beta)$, and as α and β are independent this expression has a maximum equal to $\sqrt{2} + 1$ when $\alpha + \beta = \pi/4$ and $\alpha - \beta = 0$.

The condition that the given expression have its maximum absolute value is that the points (x, y, z, w) and (a, b, c, d) shall be two points of the circle whose equations are

$$z = y \tan \alpha, \qquad w = - x \tan \alpha, \qquad x^2 + y^2 + z^2 + w^2 = 1.$$

—Editor.